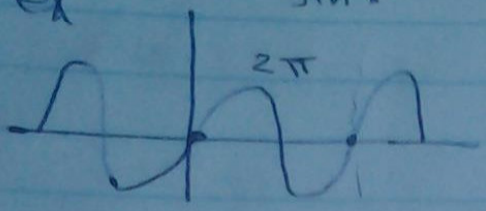
⊛ Periodic fn

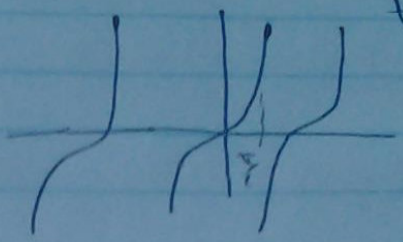
f(x) Periodic → $T = 2L$ if $f(x+T) = f(x)$

ex

sin x



tan x



$T = \pi$

⊛

$f(\alpha x) \longrightarrow \frac{T}{\alpha}$

$f\left(\frac{x}{\alpha}\right) \longrightarrow \alpha T$

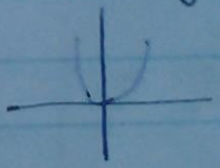
ex: $\sin x \longrightarrow 2\pi$

$\sin 2x \longrightarrow \pi$

$\sin \frac{x}{3} \longrightarrow 6\pi$

⊛ even fn

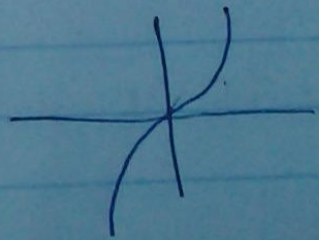
if $f(-x) = f(x)$ متماثل حول محور y



ex: x^2 or $\cos x$

⊛ odd fn

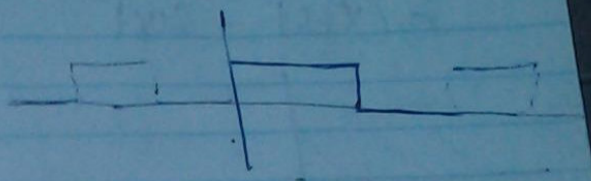
$f(-x) = -f(x)$ متماثل حول نقطة الأصل



ex: x^3

Ex: Find F.S of $f(x) = \begin{cases} 1 & 0<x<3 \\ 0 & 3<x<6 \end{cases}$

$f(x+6) = f(x)$ $\quad 2L = 6 \quad L = 3$



$$a_0 = \frac{1}{3}\int_0^3 1\,dx = 1$$

$$a_n = \frac{1}{3}\int_0^3 1\cdot\cos\frac{n\pi}{3}x\,dx = \frac{1}{3}\cdot\frac{3}{n\pi}\sin\frac{n\pi}{3}x\Big|_0^3$$

$$= \frac{1}{n\pi}(\sin n\pi - \sin 0) = 0$$

$\sin n\pi = 0$

$\cos n\pi = (-1)^n$

$$b_n = \frac{1}{3}\int_0^3 \sin\frac{n\pi}{3}x\,dx = \frac{-1}{3}\cdot\frac{3}{n\pi}\cos\frac{n\pi}{3}\Big|_0^3$$

$$= \frac{-1}{n\pi}(\cos n\pi - \cos 0) = \frac{-1}{n\pi}\left((-1)^n - 1\right) \begin{cases} 0 & n \text{ (even)} \\ \frac{2}{n\pi} & (n \text{ odd}) \end{cases}$$

$$f(x) = \frac{1}{2} + \sum_{n=1}^{\infty}\frac{1}{n\pi}\left((-1)^n - 1\right)\sin\frac{n\pi}{3}x$$

$2n \rightarrow$ even

$2n+1 \rightarrow$ odd

$$b_{2n-1} = \frac{2}{(2n-1)\pi} \qquad n = 1, 2, 3 \ldots$$

$2n-1$ هو الحد الذي يحتوي على n فقط

$$f(x) = \frac{1}{2} + \sum_{n=1}^{\infty}\frac{2}{(2n-1)\pi}\sin\frac{(2n-1)\pi}{3}x$$

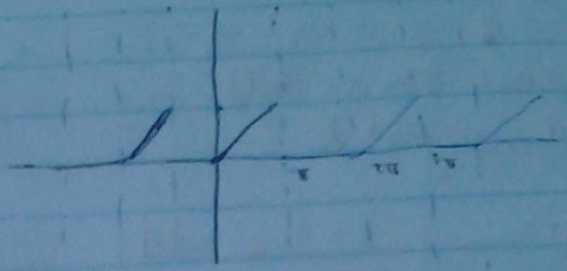
هذا هو شكل متسلسلة فورييه

$$f(x) = \begin{cases} 0 & -\pi < x < 0 \\ \frac{x}{2} & 0 < x < \pi \end{cases} \qquad f(x+2\pi) = f(x)$$

Hence use expansion to P.T

$$\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots$$

$L = \pi$



$$a_0 = \frac{1}{\pi}\int_0^{\pi} \frac{x}{2}\,dx = \pi/4$$

$$a_n = \frac{1}{\pi}\int_0^{\pi} \frac{x}{2}\cos nx\,dx$$

$$= \frac{1}{2\pi n^2}\left[(-1)^n - 1\right]$$

$\frac{x}{2}$ < $\cosh v$

$\frac{1}{2}$ → $\frac{1}{n}\sin nv$

0 → $-\frac{1}{n^2}\cosh v$

$$b_n = \frac{1}{\pi}\int_0^{\pi} \frac{x}{2}\sin nx\,dx = \frac{(-1)^{n+1}}{2n}$$

$$f(x) = \frac{\pi}{8} + \sum_1^{\infty} \frac{1}{2n^2\pi}\left((-1)^n - 1\right)\cos nx + \frac{(-1)^{n+1}}{2n}\sin nx$$

$$= \frac{\pi}{8} + \frac{-1}{\pi}\cos x + \frac{1}{2}\sin x + \frac{-1}{4}\sin 2x - \frac{1}{9}\cos 3x + \frac{1}{8}\sin 3x$$

Put $x = 0$

$$0 = \frac{\pi}{8} - \frac{1}{\pi} - \frac{1}{9\pi} - \frac{1}{25}$$

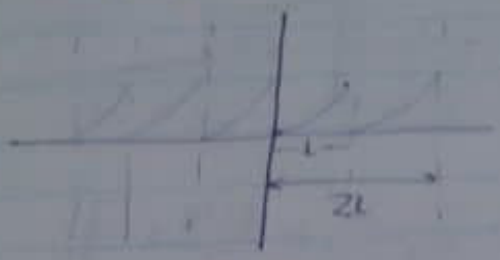
$$\frac{\pi}{8} = \frac{1}{\pi}\left(1 + \frac{1}{9} + \frac{1}{25} + \cdots\right)$$

$$\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots$$

* even Harmonic f(x) Periodic with Period 2L

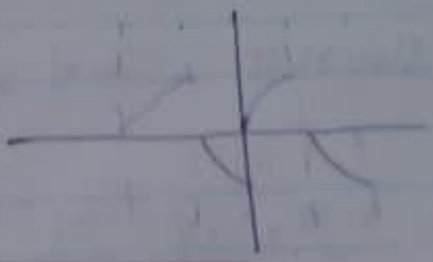
هي الدالة التي تكرر نفسها كل نفس دورة من

$$f(x+L) = f(x)$$



* odd Harmonic

$$F(x+L) = -f(x)$$



* لو الدالة ليست دورية او غير periodic فانه لا يمكن ايجاد fourier series

$$f(x) \longrightarrow 2L$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi}{L}x + b_n \sin\frac{n\pi}{L}x$$

$$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\,dx$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi}{L}x\,dx$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi}{L}x\,dx$$

$-L \longrightarrow L$

all the Period